Projective Set: How to Play

ProSet decks c/o Herman Chau, University of Washington Mathematics

Objective

Find sets of cards where, for each color, the total number of dots is **even**.

Version 1: Deal 12 cards

In the 12-card version, a set is exactly three cards.

The first player to find three cards which form a set and call out "set" takes the three cards. Three new cards are then dealt and the play continues until the deck is depleted.

If at any time the players agree there is no set among the cards, three new cards can be dealt, bringing the total number of cards on the table to 15. Other than this, new cards are not dealt out unless the number of cards on the table goes below 12.

The game ends when the deck is depleted and no more sets can be found among the cards on the table. **The player who captured the most sets is the winner.**

Version 2: Deal 7 cards

In the 7-card version, a set can be any size (3-7 cards).

7 cards are put out on the table at a time, and when a set is found (with anywhere from 3-7 cards), all the cards from the set are taken and then replaced. Points are given at the end according to how many **cards** each player captured rather than how many sets.

Mathematics of Projective Set

The cards of a Projective Set deck can be thought of as nonzero vectors in the finite vector space $\mathbb{F}_2^6$. The collection of all such vectors is the finite projective space with order 2 and dimension 5. Three cards form a set if and only if the corresponding points are collinear in that space. More generally, in the variant, *n* cards form a set if and only if the corresponding vectors add to the zero vector (in other words, those vectors are linearly dependent).

In Set, there can exist 20 cards out of the 81 without a set, but no more. In Projective Set, there can exist up to 32 out of the 63 cards with no (3-card) set.

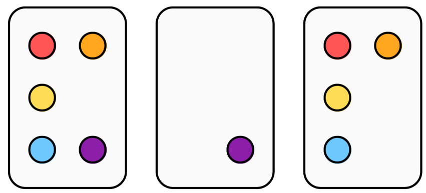
Read more: https://en.wikipedia.org/wiki/Projective_Set_(game)

Examples

$$\begin{pmatrix} \text{red} \\ \text{orange} \\ \text{yellow} \\ \text{green} \\ \text{blue} \\ \text{purple} \end{pmatrix}$$

To find the corresponding vector for a card, create a 6×1 column vector by placing a ‘1’ in each position that represents a color present on the card, and a ‘0’ in positions for colors that are not present.

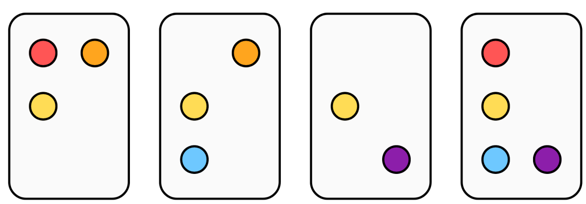
A set of 3 cards:



Corresponding vector addition:

$$\begin{pmatrix}1\\1\\1\\0\\1\\1\end{pmatrix} + \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix} + \begin{pmatrix}1\\1\\1\\0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\2\\2\\0\\2\\2\end{pmatrix} \sim \begin{pmatrix}0\\0\\0\\0\\0\\0\end{pmatrix} \quad \text{in } \mathbb{F}_2^6$$

A set of 4 cards:



Corresponding vector addition:

$$\begin{pmatrix}1\\1\\1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\1\\0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\\0\\0\\1\end{pmatrix} + \begin{pmatrix}1\\0\\1\\0\\1\\1\end{pmatrix} = \begin{pmatrix}2\\2\\4\\0\\2\\2\end{pmatrix} \sim \begin{pmatrix}0\\0\\0\\0\\0\\0\end{pmatrix} \quad \text{in } \mathbb{F}_2^6$$

A non-set (why?):